Assignment 1

Due Friday Feb 3rd, 2017

Submission Instructions: Submit solutions in a single PDF via OWL. Assignments are due at 11:59:59 pm (Eastern Time) on the date listed above. Assignments submitted more than two days late will not be accepted and a mark of zero (0) will be recorded. See the course outline for details.

1. [6 marks] Describe the language

The formal description of a DFA M is $(S = \{q_0, q_1, q_2, q_3, q_4\}, \Sigma = \{a, b, c\}, \delta, start = q_0, F = \{q_3\})$, where δ is given by the following table:

	a	b	c
q_0	q_1	q_0	q_2
q_1	q_4	q_1	q_3
q_2	q_3	q_2	q_4
q_3	q_4	q_3	q_4
q_4	q_4	q_4	q_4

(a) [4 marks] Give the state diagram of this machine.

(b) [2 marks] Using *set builder* notation, describe the language accepted by M.

2. [8 marks] Identify the regular languages

For each of the following languages state whether it is regular or not. If L_i is regular, prove it by drawing a DFA or NFA (your choice) that recognizes it. If the language is not regular, give an argument (in plain English) why there is no DFA or NFA that can recognize it:

(a) [2 marks] $L_a = \{w(a^n)w : w \in \{a, b\}, n > 0\}$

(b) [2 marks] $L_b = \{wabw : w \in \{a, b\}^*$ and $|w| > 2\}$

(c) [2 marks] $L_c = \{wwx : w, x \in \{a, b\}^*\}$ and $|x| > 3\}$

(d) [2 marks] $L_d = \{(ababab)^n : n > 0\}$

3. [10 marks] Recognizing hex integers divisible by 5

Let string $s \in \{0-9, A-F\}^*$. Let n be string s interpreted as a hexidecimal integer. Draw a DFA that accepts s if and only if:

$$n \equiv 0 \mod 5.$$

Assume $\varepsilon \not\equiv 0 \mod 5$.

4. [6 marks] Design NFAs

Let $\Sigma = \{0, 1, 2\}$. Draw an NFA recognizing each of the following languages:

(a) [2 marks] The set of strings that contain a single 1,

(b) [2 marks] The set of strings with an even length n where $n > 2$

(c) [2 marks] The set of strings that contain no consecutive digits (i.e., a 0 cannot follow a 0, etc).

5. [10 marks] An NFA in an Economy of States

Let $s \in \Sigma = \{a\}^*$. Let $|s|$ denote the length of string s. Construct a finite automaton in less than a *dozen* states that recognizes language:

$$L = \{s : \gcd(|s|, 300) \neq 1\},$$

where $\gcd(a, b)$ denotes the greatest common divisor between a, b.

6. [10 marks] Prove Finite Languages are Regular

We say a language L is *finite* if L contains a finite number of strings. Using induction, prove all finite languages are regular.