

SE3310 Pumping Lemma Study Guide

Pumping Lemma Recap

Recall the pumping lemma for regular languages states that for all strings in L longer than the pumping length (p), there exists *some* way to cut s into 3 parts xyz such that:

1. For all $i \geq 0$, $xy^iz \in L$
 - Explanation: the middle part y can be pumped (i.e., repeated arbitrarily many times, or even removed) and the resulting string is also in L
2. $|y| > 0$
 - Explanation: y , the loop portion, is non-empty. If y was empty, then pumping y wouldn't do anything!
3. $|xy| \leq p$
 - Explanation: p is defined to be the length after which a loop must occur in the corresponding DFA. x represents the portion of the string generated on the way leading up to the entrance of the loop, and y represents the portion of the string generated by the loop itself. Thus xy represents the string generated by a path through the DFA that contains a loop, and thus $|xy| \leq p$. This property can be useful in some languages as a means to make the proof simpler. It can allow the prover to reducing the number of possible cases for what y can contain.

Proof Steps

When performing the pumping lemma for regular languages, all proofs proceed with similar steps. Suppose we are asked to prove a language L is not regular.

1. Toward a contradiction assume L is regular
2. Let p be the pumping length
3. Pick a string s such that $s \in L$ and $|s| \geq p$
 - Note: The choice of s is left up to the prover, so pick a string that you think will

lend itself well to forming the contradiction

4. We must now examine all possible ways to divide s into 3 parts. Let $s = xyz$. We have to consider all possible cases for what might be contained in y .
 - Tip: From the pumping lemma we already know 2 things about y right out of the gate: $|y| \geq 0$ and $|xy| \leq p$ and thus $|y| \leq p$.
5. For each case identified in the previous step we must find our contradiction. That is, we must find a value $i \geq 0$ such that $xy^iz \notin L$.
 - Note: we only need to find one value for i in order to obtain the contradiction (since item 1 of the pumping lemma says it should hold for all $i \geq 0$).
6. Assuming we could successfully find a contradiction (step 5) for all possible cases (step 4) we conclude there is *no* way to divide s into 3 parts xyz such that y can be pumped. Therefore we conclude L is not regular.

An Example

Let $L = a^n b^n : n > 0$. Prove L is not regular.

Proof: We can directly follow the 6 proof steps outlined above.

1. Assume L is regular
2. Let p be the pumping length
3. Let $s = a^p b^p$. Clearly $s \in L$ and $|s| = 2p$ and thus $|s| \geq p$
4. Consider the case where y contains only a 's. There are no other cases since s is a string consisting of p a 's followed by p b 's, and as per the pumping lemma $|xy| \leq p$, meaning the xy portion of the string cannot include any b 's.
5. Let $s = xyz$. Suppose we pump y up. Let $s_2 = xy^2z$, i.e., $xyyz$. Since y is non-empty, it means we added some non-zero number of 0 's to the string. That means s_2 contains *more* 0 's than 1 's. But L is the language of strings that contain 0 's followed by an *equal* number of 1 's. Therefore $s_2 \notin L$.
6. According to the pumping lemma s_2 should be in L . This is a contradiction. Therefore L is not regular.